Trigonometric Identities:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin\alpha\ \sin\beta = \tfrac{1}{2}\cos(\alpha-\beta) - \tfrac{1}{2}\cos(\alpha+\beta)$$
$$\cos\alpha\ \cos\beta = \tfrac{1}{2}\cos(\alpha+\beta) + \tfrac{1}{2}\cos(\alpha-\beta)$$
$$\sin\alpha\ \cos\beta = \tfrac{1}{2}\sin(\alpha+\beta) + \tfrac{1}{2}\sin(\alpha-\beta)$$

Euler's Identity:

$$e^{j\theta} = \cos\theta\ +\ j\ \sin\theta \qquad \cos\alpha = \frac{1}{2}\left(e^{j\alpha} + e^{-j\alpha}\right) \qquad \sin\alpha = \frac{1}{2j}\left(e^{j\alpha} - e^{-j\alpha}\right)$$

Rectangular Pulse Function:

$$\Pi(t) = \textbf{rect}(t) = \begin{cases} 1, & |t| < \frac{1}{2}; \\ \frac{1}{2}, & |t| = \frac{1}{2}; \\ 0, & |t| > \frac{1}{2}. \end{cases}$$

Triangular Pulse Function:

$$\Delta(t) = \begin{cases} 1 - 2|t|, & |t| < \frac{1}{2}; \\ 0, & |t| > \frac{1}{2}. \end{cases}$$

Table 3.2 Properties of Fourier Transform Operations

Operation	$g(t)$	$G(f)$
Superposition	$g_1(t) + g_2(t)$	$G_1(f) + G_2(f)$
Scalar multiplication	$kg(t)$	$kG(f)$
Duality	$G(t)$	$g(-f)$
Time scaling	$g(at)$	$\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$
Time shifting	$g(t-t_0)$	$G(f)\,e^{-j2\pi f t_0}$
Frequency shifting	$g(t)\,e^{j2\pi f_0 t}$	$G(f-f_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(f)G_2(f)$
Frequency convolution	$g_1(t)g_2(t)$	$G_1(f) * G_2(f)$
Time differentiation	$\frac{d^n g(t)}{dt^n}$	$(j2\pi f)^n G(f)$
Time integration	$\int_{-\infty}^{t} g(x)dx$	$\frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\delta(f)$

Table 3.1 Short Table of Fourier Transforms

$g(t)$	$G(f)$
$e^{-at}u(t)$	$[a+j2\pi f]^{-1}, \quad a>0$
$e^{at}u(-t)$	$[a-j2\pi f]^{-1}, \quad a>0$
$e^{-a\|t\|}$	$2a\,[a^2+(2\pi f)^2]^{-1}, \quad a>0$
$te^{-at}u(t)$	$[a+j2\pi f]^{-2}, \quad a>0$
$t^n e^{-at}u(t)$	$n!\,[a+j2\pi f]^{-(n+1)}, \quad a>0$
$\delta(t)$	1
1	$\delta(f)$
$e^{j2\pi f_0 t}$	$\delta(f-f_0)$
$\cos 2\pi f_0 t$	$\frac{1}{2}\big[\delta(f+f_0)+\delta(f-f_0)\big]$
$\sin 2\pi f_0 t$	$\frac{j}{2}\big[\delta(f+f_0)-\delta(f-f_0)\big]$
$u(t)$	$\frac{1}{2}\delta(f)+[j2\pi f]^{-1}$
$\mathrm{sgn}(t)$	$2[j2\pi f]^{-1}$
$\cos(2\pi f_0 t)\,u(t)$	$\frac{1}{4}\big[\delta(f-f_0)+\delta(f+f_0)\big]+j2\pi f\big[(2\pi f_0)^2-(2\pi f)^2\big]^{-1}$
$\sin(2\pi f_0 t)\,u(t)$	$\frac{1}{4j}\big[\delta(f-f_0)-\delta(f+f_0)\big]+2\pi f_0\big[(2\pi f_0)^2-(2\pi f)^2\big]^{-1}$
$e^{-at}\sin(2\pi f_0 t)\,u(t)$	$2\pi f_0\big[(a+j2\pi f)^2+(2\pi f_0)^2\big]^{-1}, \quad a>0$
$e^{-at}\cos(2\pi f_0 t)\,u(t)$	$\big[a+j2\pi f\big]\big[(a+j2\pi f)^2+(2\pi f_0)^2\big]^{-1}, \quad a>0$
$\Pi\big(\frac{t}{\tau}\big)$	$\tau\,\mathrm{sinc}(\pi f\tau)$
$2B\,\mathrm{sinc}(2\pi Bt)$	$\Pi\big(\frac{f}{2B}\big)$
$\Delta\big(\frac{t}{\tau}\big)$	$\frac{\tau}{2}\,\mathrm{sinc}^2\Big(\frac{\pi f\tau}{2}\Big)$
$B\,\mathrm{sinc}^2(\pi Bt)$	$\Delta\big(\frac{f}{2B}\big)$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$f_0\sum_{n=-\infty}^{\infty}\delta(f-nf_0), \quad f_0=1/T_0$
$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-2(\sigma\pi f)^2}$

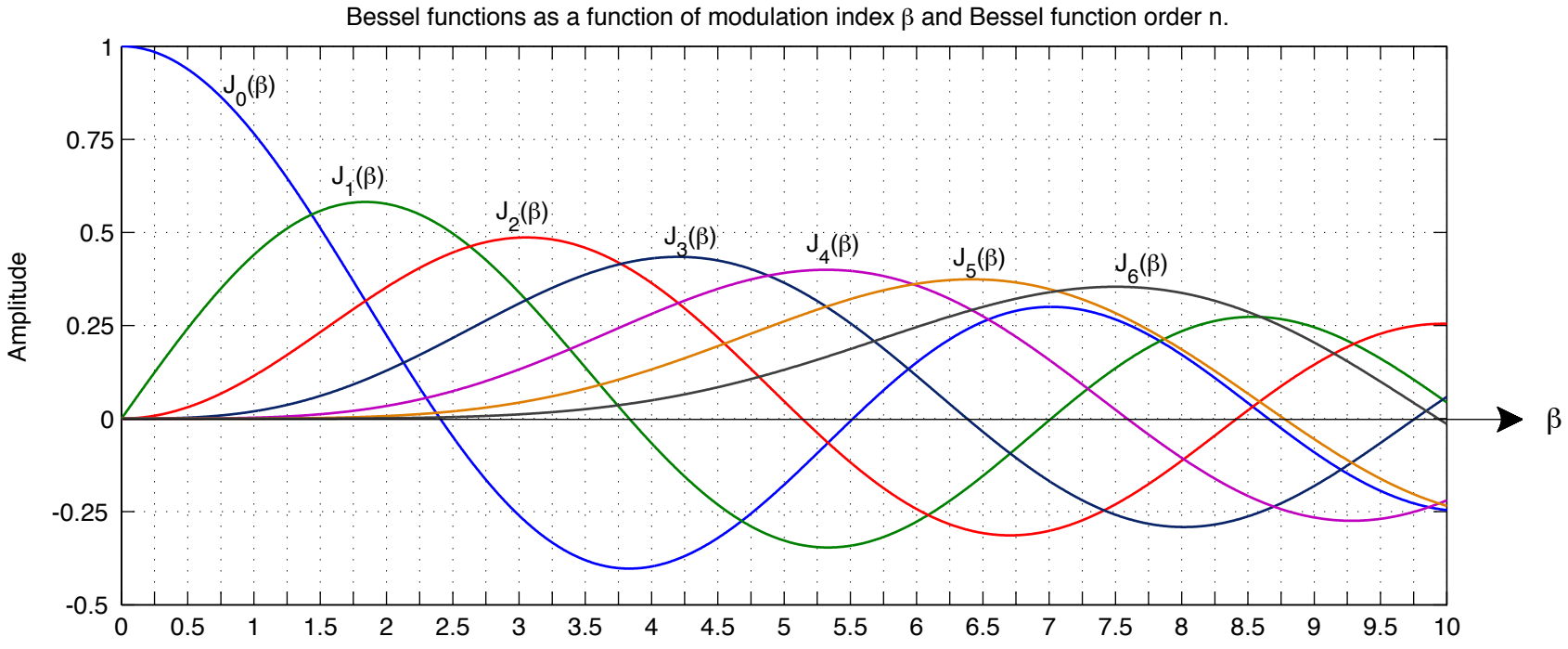
Bessel functions as a function of modulation index β and Bessel function order n.
Amplitude
β
J0(β)
J1(β)
J2(β)
J3(β)
J4(β)
J5(β)
J6(β)
1
0.75
0.5
0.25
0
-0.25
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10

Table of Bessel Functions as a function of modulation index β and Bessel function order n.

		$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$	$J_3(\beta)$	$J_4(\beta)$	$J_5(\beta)$	$J_6(\beta)$	$J_7(\beta)$	$J_8(\beta)$	$J_9(\beta)$
β	0.0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.2	0.99	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.4	0.96	0.20	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.6	0.91	0.29	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.8	0.85	0.37	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	1.0	0.77	0.44	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	1.2	0.67	0.50	0.16	0.03	0.01	0.00	0.00	0.00	0.00	0.00
	1.4	0.57	0.54	0.21	0.05	0.01	0.00	0.00	0.00	0.00	0.00
	1.6	0.46	0.57	0.26	0.07	0.01	0.00	0.00	0.00	0.00	0.00
	1.8	0.34	0.58	0.31	0.10	0.02	0.00	0.00	0.00	0.00	0.00
	2.0	0.22	0.58	0.35	0.13	0.03	0.01	0.00	0.00	0.00	0.00
	2.2	0.11	0.56	0.40	0.16	0.05	0.01	0.00	0.00	0.00	0.00
	2.4	0.00	0.52	0.43	0.20	0.06	0.02	0.00	0.00	0.00	0.00
	2.6	-0.10	0.47	0.46	0.24	0.08	0.02	0.01	0.00	0.00	0.00
	2.8	-0.19	0.41	0.48	0.27	0.11	0.03	0.01	0.00	0.00	0.00
	3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	0.00	0.00	0.00
	3.2	-0.32	0.26	0.48	0.34	0.16	0.06	0.02	0.00	0.00	0.00
	3.4	-0.36	0.18	0.47	0.37	0.19	0.07	0.02	0.01	0.00	0.00
	3.6	-0.39	0.10	0.44	0.40	0.22	0.09	0.03	0.01	0.00	0.00
	3.8	-0.40	0.01	0.41	0.42	0.25	0.11	0.04	0.01	0.00	0.00
	4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	0.00	0.00
	4.2	-0.38	-0.14	0.31	0.43	0.31	0.16	0.06	0.02	0.01	0.00
	4.4	-0.34	-0.20	0.25	0.43	0.34	0.18	0.08	0.03	0.01	0.00
	4.6	-0.30	-0.26	0.18	0.42	0.36	0.21	0.09	0.03	0.01	0.00
	4.8	-0.24	-0.30	0.12	0.40	0.38	0.23	0.11	0.04	0.01	0.00
	5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	0.01
	5.2	-0.11	-0.34	-0.02	0.33	0.40	0.29	0.15	0.07	0.02	0.01
	5.4	-0.04	-0.35	-0.09	0.28	0.40	0.31	0.18	0.08	0.03	0.01
	5.6	0.00	-0.33	-0.15	0.23	0.39	0.33	0.20	0.09	0.04	0.01
	5.8	0.09	-0.31	-0.20	0.17	0.38	0.35	0.22	0.11	0.05	0.02
	6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02